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Master Thesis Offer:

Spectral Neural Networks for Physics-informed Machine Learning

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Motivations and Description

The dynamics of many physical systems can be described using a *Partial Differential Equation* (PDE) where the temporal evolution of a physical quantity is given by a combination of its spatial derivatives. Classical examples include the *Heat Equation* for the diffusion of heat in a space, or the *Navier-Stokes Equation* for the velocity of a fluid. *Physics-Informed Neural Networks* (PINNs) have been proposed to learn a solution of such a PDE in the form of a neural network mapping spatio-temporal coordinates to the value of the solution at this input location [1]. Unfortunately, PINNs are hard to optimize and underperform compared to state of the art numerical methods.

The goal of this internship is to explore a new neural architecture mapping time and spatial frequencies to spectral components over time of the solution, hence **learning a Fourier transform of the solution**. This work will involve: i) implementing the neural network architecture, ii) developping and implementing a new loss to train the neural network, and iii) evaluate its efficacy compared to PINNs and state of the art numerical methods. In particular, we will apply it to 3 physical settings: the diffusion of heat following the heat equation, fluid velocity following the Navier-Stokes equation, and laser-matter interactions modelled with Swift-Hohenberg equation.

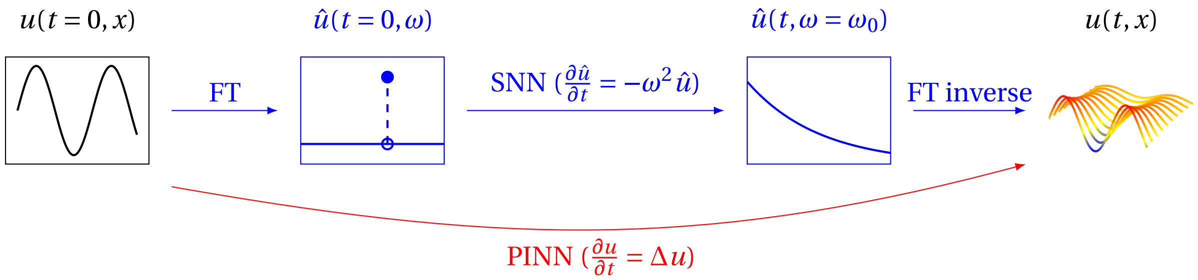

Figure 1: Difference between PINNs and a *Spectral Neural Network* (SNN) with the example of a heat equation solution u. The system starts in the state $u(t=0,x)$ giving the temperature for all locations x at the initial time $t=0$. The *Fourier Transform* (FT) maps its input to the input's spectral representation. The goal of the internship is to implement the blue approach.

Expected Results

- Develop a novel loss function for physics-informed spectral neural networks
- Implement both the architecture and the loss using DeepXDE
- Run experiments solving various PDEs and compare the efficacy of the approach to:
  - PINNs using DeepXDE
  - State of the art numerical methods

Continuation of the work with a PhD can be discussed.

References

[1] M. Raissi, P. Perdikaris, and G. Karniadakis. Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations. *Journal of Computational Physics*, 378:686–707, 2019.